Research Article

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Effect of Inerter in Traditional and Variant Dynamic Vibration Absorbers for One Degree-of-Freedom Systems Subjected to Base Excitations

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Abstract: In this paper, closed-form optimal parameters of inerter-based variant dynamic vibration absorber (variant IDVA) coupled to a primary system subjected to base excitation are derived based on classical fixed-points theory. The proposed variant IDVA is obtained by adding an inerter alone parallel to the absorber damper in the variant dynamic vibration absorber (variant DVA). A new set of optimum frequency and damping ratio of the absorber is derived, thereby resulting in lower maximum amplitude magnification factor than the inerter-based traditional dynamic vibration absorber (traditional IDVA). Under the optimum tuning condition of the absorbers, it is proved both analytically and numerically that the proposed variant IDVA provides a larger suppression of resonant vibration amplitude of the primary system subjected to base excitation. It is demonstrated that adding an inerter alone to the variant DVA provides 19% improvement in vibration suppression than traditional IDVA when the mass ratio is less than 0.2 and the effective frequency bandwidth of the proposed IDVA is wider than the traditional IDVA. The effect of inertance and mass ratio on the amplitude magnification factor of traditional and variant IDVA is also studied.

Keywords: Dynamic vibration absorber, Minimax, Inerter, Traditional IDVA, Variant IDVA

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1 Introduction

A tuned-mass damper, or a dynamic vibration absorber (DVA), is a passive vibration device used to reduce resonant vibration. The concept of undamped DVA was introduced by [9] and it is useful in a narrow range of frequencies very close to the natural frequency of the DVA. [16] introduced the damped DVA which is useful over a wide frequency band. It is also known as the traditional DVA in which a spring and damping element are arranged in parallel. [16] first deduced that all frequency response curves of the undamped primary system pass through two invariant points independent of absorber damping when a harmonic force is applied to the primary system. Following this characteristic, the desired optimum value of the absorber's resonance frequency can be found when the heights of two points are equal. [2] developed the mathematical theory known as fixed-points theory and showed that the optimum absorber damping ratio can be detected by making the height of the fixed points to the maximum. The optimal tuning ratio and damping ratio of the traditional DVA determined by using the fixed-point method are not exact, because some approximations are taken when optimal parameters are derived. The optimum parameters were introduced by Den Hartog through his well-known textbook *Mechanical Vibrations*, and now we all know these expressions [10]. However, [15] proposed the exact solutions and compared those with the results given by [10]; they found that both optimal tuning ratio and damping ratio presented by Den Hartog were very close to the exact solutions. When the primary system takes into account damping, it is difficult to obtain analytical solutions for the optimum parameters of the standard DVA. [1] presented a series of analytical solutions for the DVA attached to damped primary systems by minimizing the maximum amplitude magnification factor of damped primary systems. For damped primary system, a number of studies have focused on the numerical solutions. These include, but are not limited to, numerical optimization schemes

[13, 18], frequency locus method [22] and min–max criteria [3, 8, 17]. A non-traditional type DVA, also called a variant DVA, was introduced by [19] in which the absorber damper is connected to the rigid frame. For undamped primary system, the design procedure for traditional DVAs has also been extended to variant DVAs and several attempts have been made to find optimal parameters analytically [6, 7, 14], and for damped primary systems, analytical and numerical solutions are available in [13].

Recently, an inerter-based DVA has attracted many researchers. It has been successfully used in Formula-one vehicle suspension systems initially and after that applied to various mechanical systems mainly, including vehicle suspensions and vibration suppression. It is a two-terminal mechanical device introduced by [21], which has the property that the applied force at its two terminals is directly proportional to the relative acceleration between the two terminals, and the constant of proportionality is called inertance with a unit of kilogram. [5] investigated the influence of inerter on the natural frequencies of vibrating systems and found that the inerter can reduce the natural frequencies of vibrating systems. [12] derived the optimal parameters of traditional IDVA for undamped primary system using fixed-points theory and also concluded that adding a single inerter alone to the traditional DVA, whether in parallel connection or in series connection, provides no improvement in the H_∞ optimization. [11] investigated the damping performance of inerter-based isolator and compared the optimal parameters of inerter-based isolator with traditional DVA. [20] investigated the inerter-based DVA added on the body mass and concluded that the new suspension structure called inerter-based suspension can effectively improve the damping performance of the suspension system. [4] investigated how additional damping and inerter introduced by supplementary devices influence the system dynamics and compared the effects caused by the additional damper and inerter.

From the previous studies, it is clear that extensive studies have been carried out on optimization of traditional and variant DVAs. However, optimization of variant IDVA for damped and undamped primary systems has not been studied extensively. To the author's knowledge, there is no research report found in literature on this topic. The primary focus of this present work is to derive the closed-form analytical expression for optimal parameters of variant IDVA excited by ground motion using fixed-points theory, whereas for damped primary system, numerical optimization techniques are used. The amplitude magnification factor variant IDVA is compared with traditional IDVA and it is proved that the proposed IDVA provides a larger suppression of resonant vibration amplitude of the primary system excited by ground motion than the traditional IDVA, even though the inerter alone is coupled to the absorber damper in a parallel manner. All these constitute the main contributions of this paper.

This paper is organized as follows: Mathematical model and equation of motion are introduced in Section 2. Optimal solution to variant IDVA based on fixed-points theory is explained in Section 3. The minimax optimization problem formulation and numerical simulation, and the comparison of traditional and variant IDVAs are explained in Sections 4 and 5, respectively. Section 6 provides detailed analysis and discussions of results. Section 7 summarizes important findings of the paper.

2 Methodology

2.1 Mathematical model and equation of motion

Inerter-based traditional and variant DVA is obtained by adding an inerter alone parallel to traditional DVA and variant DVA. Figure 1 shows traditional and variant IDVA systems coupled to damped primary system subjected to base excitation.

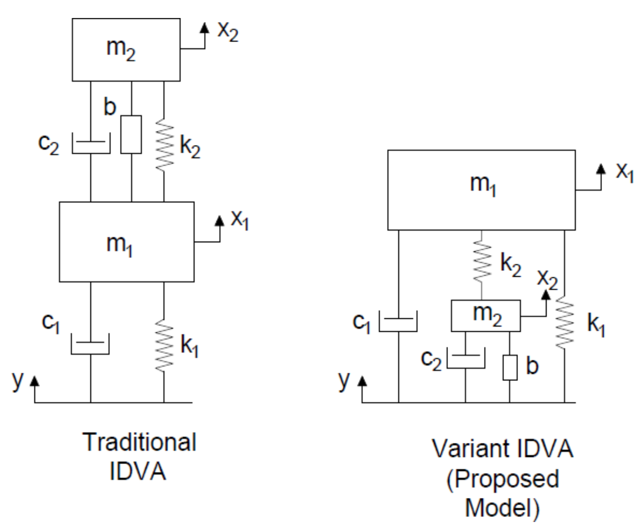


Figure 1: Types of various inerter-based dynamic vibration absorbers for damped primary system: (a) traditional IDVA; (b) variant IDVA.

The equation of motion of the proposed variant IDVA is given by

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = c_1\dot{y} + k_1y \qquad (1)$$

$$m_2\ddot{x}_2 + b\ddot{x}_2 + c_2\dot{x}_2 + k_2(x_2 - x_1) = b\ddot{y} + c_2\dot{y} \qquad (2)$$

where m_1 and m_2 are the masses of the primary and absorber systems, x_1 and x_2 are the displacement of the primary and absorber systems, k_1 and k_2 are the linear spring constants of the primary and absorber systems, and c_1 and c_2 are the viscous damping coefficients of the primary and absorber systems, respectively; and b is the inertance. Assuming a harmonic disturbing input $y = Ye^{-j\omega t}$.

2.2 Dynamic amplitude magnification factor for undamped primary system

For traditional and variant undamped DVA, the fixed-point theory is commonly used to obtain the optimal parameters. The absorber frequency and damping ratio of traditional IDVA excited by ground motion is equal to the optimal parameters of traditional IDVA which is derived by [12]. They concluded that there is no improvement in the H_∞ optimization by adding inerter alone to the traditional IDVA. In this section, the absorber and frequency damping ratios of variant IDVA are derived by using fixed-points theory, because there always exist two invariant (fixed) points with respect to damping ratio of the variant IDVA. The non-dimensional amplitude magnification factor of the proposed IDVA excited by ground motion is as follows:

$$G = \left|\frac{X_1}{Y}\right| = \frac{\sqrt{(f^2 - r^2 - \delta f^2\mu r^2 - \delta r^2)^2 + (2\xi_2 f^3\mu r + 2\xi_2 fr)^2}}{\sqrt{A}} \qquad (3)$$

where

$$A = \left(r^4 + f^2 + \delta r^4 - r^2 - \delta f^2\mu r^2 - \delta r^2 - f^2\mu r^2 - f^2r^2\right)^2 + \left(2\xi_2 f\left(f^2\mu + 1\right)r - 2\xi_2 fr^3\right)^2$$

To obtain the non-dimensional amplitude magnification factor of the proposed IDVA model, the following dimensionless parameters are used:

$$\omega_i = \sqrt{\frac{k_i}{m_i}}, \quad f = \frac{\omega_2}{\omega_1}, \quad \mu = \frac{m_2}{m_1}, \quad \xi_i = \frac{c_i}{2\sqrt{k_i m_i}}, \qquad (4)$$
$$r = \frac{\omega}{\omega_1}, \quad \delta = \frac{b}{m_2}, \quad i = 1, 2$$

To exhibit the characteristics, the results in numerous cases of absorber damping ratio ξ_2 are given in Figure 2 under m_1 = 1 kg, μ = 0.1, δ = 0.1, f = 1.1189. It is obviously seen that there exist two common points (P and Q) on all the amplitude magnification factor of primary system response curves, where the amplitude magnification factor of primary system is not influenced by the absorber damping ratio. These points are referred to as the fixed points or invariant points.

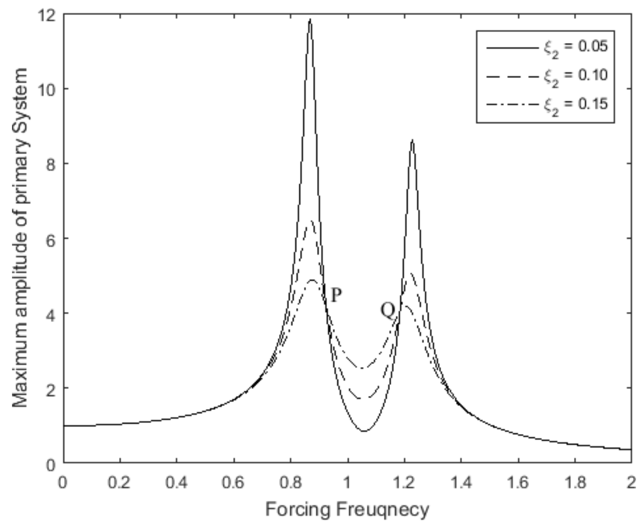


Figure 2: The amplitude magnification factor of variant IDVA with primary system where $\mu = 0.1, \delta = 0.1$ and various values of absorber damping ratio ($\xi_2 = 0.05, 0.10, 0.15$).

The optimum condition of variant IDVA can be attained by fine-tuning the responses at P and Q to the equal level (optimum frequency), and meanwhile making P and Q the maximum points on the response curves (optimum damping). This design rule is the eminent fixed-points theory [10]. In the following, this theory will be utilized to develop the optimum design of variant IDVA. To find the fixed points P and Q, Den Hartog expressed Equation (3) in the form

$$G = \sqrt{\frac{A\xi^2 + B}{C\xi^2 + D}} \qquad (5)$$

where

$$A = (2\xi_2 f^3\mu r + 2\xi_2 fr)^2, \quad B = (f^2 - r^2 - \delta f^2\mu r^2 - \delta r^2)^2,$$
$$C = \left(2\xi_2 f\left(f^2\mu + 1\right)r - 2\xi_2 fr^3\right)^2$$
$$D = \left(r^4 + f^2 + \delta r^4 - r^2 - \delta f^2\mu r^2 - \delta r^2 - f^2\mu r^2 - f^2r^2\right)^2$$

which is totally independent of absorber damping ratio (ξ_2) if $\frac{A}{C} = \frac{B}{D}$. Following the same procedure, we solve the equation

$$\frac{(f^2\mu + 1)^2}{(f^2\mu + 1 - r^2)^2} = \frac{(f^2 - r^2 - \delta f^2\mu r^2 - \delta r^2)^2}{(r^4 + f^2 + \delta r^4 - r^2 - \delta f^2\mu r^2 - \delta r^2 - f^2\mu r^2 - f^2r^2)^2}$$

The solutions are

$$r_1 = \frac{(-H)(J-I)^{\frac{1}{2}}}{2}; \quad r_2 = \frac{(-H)(J+I)^{\frac{1}{2}}}{2}$$

where

$$H = \frac{\sqrt{-(4\delta f^2\mu + 2f^2\mu + 4\delta + 4)}}{(2\delta f^2\mu + f^2\mu + 2\delta + 2)}$$
$$I = (-2\delta f^4\mu^2 - f^4\mu^2 - f^4\mu - 4\delta f^2\mu - 3f^2\mu - 2f^2 - 2\delta - 2)$$
$$J = (4\delta^2 f^8\mu^4 + 4\delta f^8\mu^4 + 4\delta f^8\mu^3 + f^8\mu^4 + 16\delta^2 f^6\mu^3 + 2f^8\mu^3 + 20\delta f^6\mu^3 + f^8\mu^2 + 6f^6\mu^3 + 24\delta^2 f^4\mu^2 + 2f^6\mu^2 + 36\delta f^4\mu^2 + 4f^6\mu - 12\delta f^4\mu + 13f^4\mu^2 + 16f^2\delta^2\mu - 8f^4\mu + 28\delta f^2\mu + 4f^4 - 8\delta f^2 + 12f^2\mu + 4\delta^2 - 8f^2 + 8\delta + 4)^{\frac{1}{2}}$$
$$K = (-2\delta - 2)$$

The ordinates of points P and Q can be found by approximating $\xi \to \infty$ in the equation

$$G = \sqrt{\frac{(f^2\mu + 1)}{(f^2\mu + 1 - r^2)}} \tag{6}$$

and the optimum value for f_{opt} is obtained by equating $G(r_1)$ and $G(r_2)$

$$\frac{(f^2\mu + 1)}{(f^2\mu + 1 - r^2)} = -\frac{(f^2\mu + 1)}{(f^2\mu + 1 - r^2)}$$

This gives

$$f_{opt} = \sqrt{\frac{-(8\mu\delta + 2\sqrt{-8\delta\mu + \mu^2 - 4\mu + 4} + 6\mu - 4)}{4\mu(2\delta\mu + \mu - 1)}} \tag{7}$$

Substituting r_1 and f_{opt} in Equation (6) gives the common ordinate

$$G_{opt} = G(r_1) = G(r_2) = \frac{L}{(M+N)^{\frac{1}{2}}} \tag{8}$$

where

$$L = \frac{-1(\sqrt{4 + \mu^2 - 8\delta\mu - 4\mu} + \mu)}{4} \cdot \left[(2\delta\sqrt{4 + \mu^2 - 8\delta\mu - 4\mu} + \sqrt{4 + \mu^2 - 8\delta\mu - 4\mu} - 2\delta\mu - \mu + 2)\sqrt{-2\mu})\right]$$

$$M = ((16\delta^3 + 24\delta^2 + 8\delta - 1)\mu^2 + (16\delta^2 - 12\delta)\mu + 4\delta) \cdot \sqrt{4 + \mu^2 - 8\delta\mu - 4\mu} - 8\delta$$

$$N = (32\delta^4 + 80\delta^3 + 72\delta^2 + 24\delta + 1)\mu^3 + (-64\delta^3 - 96\delta^2 - 40\delta - 2)\mu^2 + (40\delta^2 + 28\delta)\mu$$

In Equation (7), the optimum tuning ratio condition was inferred. In order to find the optimum damping ratio to make points P and Q the maximum on the amplitude magnification factor of primary system frequency response curve, now we apply Brock's [2] approach to find the optimum damping ratios at points P and Q (*i.e.* ξ_P and ξ_Q). Taking average of both the values, finally we obtain the absorber damping ratio optimal value when inertance-to-mass ratio is zero:

$$\xi_2 = \frac{(\mu^7(\mu-3)(\mu-2)(\mu-1))}{48\left((Y-X)\left(\frac{1}{3}\sqrt{(\mu-2)^2} - \frac{2}{3} + \mu\right)(Y+X)\right)^{\frac{1}{2}}} \tag{9}$$

where

$$X = \left(\frac{1}{8}\mu^2(\mu-2)^2\left(\mu - \sqrt{(\mu-2)^2} - 2\right) \cdot \sqrt{\mu^2\left(\mu - \sqrt{(\mu-2)^2} - 2\right)}\right)$$

$$Y = \sqrt{-2}\left(\left(\frac{3}{4}\mu^{9/2} - \frac{3}{2}\mu^{7/2} - \frac{1}{8}\mu^{11/2} + \mu^{5/2}\right)\sqrt{(\mu-2)^2} - \mu^{11/2} + \frac{\mu^{13/2}}{8} + 3\mu^{9/2} + 2\mu^{5/2} - 4\mu^{7/2}\right)$$

Apparently, with the approximation involved in Equation (9), one cannot accurately make points *P* and *Q* as the maximum points of the amplitude magnification factor of the proposed variant IDVA primary system response curve. Figure 3 shows the amplitude magnification factor of the proposed variant IDVA primary system response curves under the optimum tuning condition derived herein. Equations (7)–(9) clearly show that the optimal parameters and

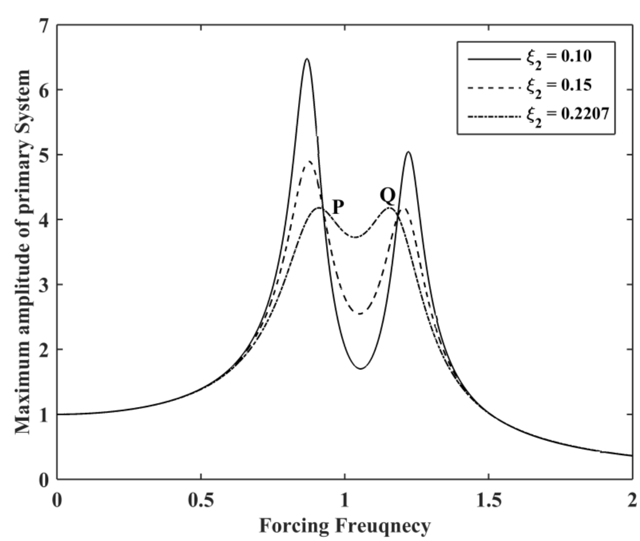


Figure 3: Amplitude magnification factor of variant IDVA primary system where $\mu = 0.1$, $f = 1.1189$, $\delta = 0.1$ and for various values of absorber damping ($\xi_2 = 0.2207$, $\xi_2 = 0.10$, $\xi_2 = 0.15$).

Table 1: Comparison of Numerical Results of IDVAs with Approximate Solutions.

Inerter-Based Dynamic Vibration Absorber	Variables/ Function	Approximate Solution when $\xi_1 = 0$	Present Study Numerical solution when $\xi_1 = 0$
	$\xi_{2,opt}$	0.185	0.186
Traditional IDVA $\mu = 0.1$, $\delta = 0.1$	f_{opt}	0.957	0.957
	G	4.817	4.822
	$\xi_{2,opt}$	0.210	0.220
Variant IDVA $\mu = 0.1$, $\delta = 0.1$	f_{opt}	1.115	1.118
	G	4.133	4.179

amplitude magnification factor of the primary system of variant IDVAs will be equal to the optimal parameters of variant DVA excited by ground motion, which is derived by [23] when the inertance-to-mass ratio (δ) of variant IDVA becomes zero. The optimal parameters of traditional IDVA subjected to base excitations are derived analytically by using fixed-points theory and are shown:

$$f_{opt} = \frac{\sqrt{\delta(1+\mu)+1}}{1+\mu} \tag{10}$$

$$\xi_2 = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{11}$$

and the optimal height at the two fixed points is

$$G_{opt} = \sqrt{\frac{2\delta(\mu+1)+\mu+2}{\mu}} \tag{12}$$

2.3 Minimax optimization formulation

In most systems, the frequencies of the external disturbances are not known exactly. An ideal vibration absorber will perform well over a range of possible external forcing frequencies. [17] has developed a method which uses Chebyshev's min–max criterion to arrive at the optimal parameters of a vibration absorber. The minimax optimization finds the values of design variables which minimize the normalized displacement of the primary system over the frequency ranges. According to [18], it has been assumed that ξ_1 and μ are independent parameters and the remaining parameters that have to be optimized are ξ_2 and f.

The optimization problem proposed in this paper can be constituted as the minimax problem:

$$\min_{\xi_2, f} \max_{(r_l \le r \le r_u)} G \tag{13}$$

The solution to Equation (13) will be the ξ_2 and f which will minimize the maximum over the domain of interest of frequency ratio range r.

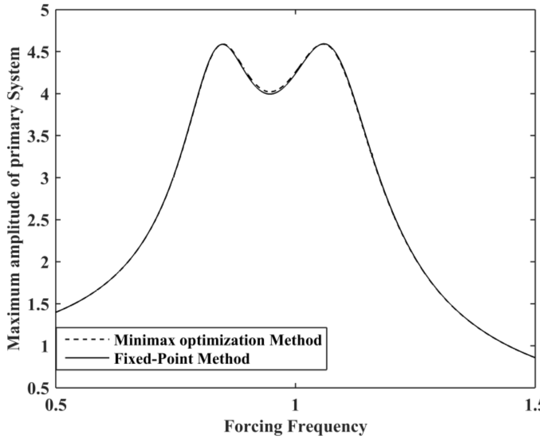
2.4 Numerical results and comparison

For traditional and variant DVAs in which $\xi_1 = 0$, a closed-form analytical solution was given by [10] and [9]. For traditional IDVAs in which $\xi_1 = 0$, a closed-form analytical solution of frequency and absorber damping ratio was given by [12]. For variant IDVA excited by ground motion, the closed-form analytical solution is presented in this paper. For undamped system, the numerical results are compared with analytical solutions derived for inerter-based traditional and variant IDVAs excited by ground motion and are shown in Table 1. The same values are plotted in Figure 4 under the following values $m_1 = 1$ kg, $\mu = 0.1$, $\xi_1 = 0$, $\omega_1 = 1$ rad/sec, and $\delta = 0$, respectively. Close agreement between the two values are observed.

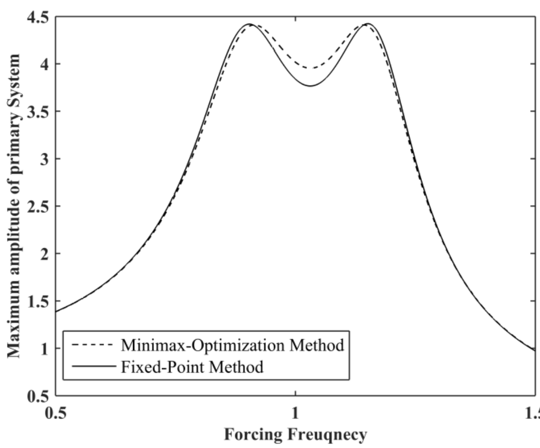
To get an optimum variant IDVA excited by ground motion, a larger frequency and damping ratio are needed. It gives better vibration suppression than traditional IDVA which is evident with the obtained G value that is smaller when compared to the same obtained for traditional IDVA.

3 Analysis and discussion of results

Figure 5 shows the comparison between traditional and variant IDVAs for different values of inertance-to-mass ratio (δ) under the following conditions $m_1 = 1$ kg, $\mu = 0.1$, $\xi_1 = 0$, $\omega_1 = 1$ rad/sec, respectively. It is clearly shown that the amplitude magnification factor of primary system increases with increase in inertance-to-mass ratio as shown in Figure 5(a) and in the case of variant IDVA, the amplitude magnification factor of primary system decreases with increase in inertance-to-mass ratio as shown in Figure 5(b). As a result, it is sufficient to conclude that the proposed variant IDVA provides significant improvement in the suppression of vibration of the primary system by

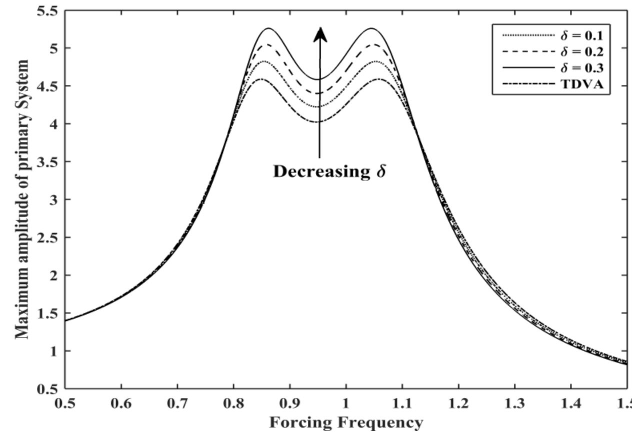


(a)

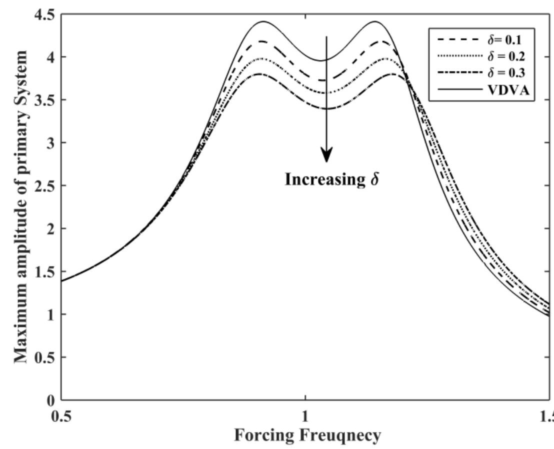


(b)

Figure 4: Comparison of amplitude magnification factor of primary system when $\xi_1 = 0$, $\delta = 0$: (a) traditional IDVA; (b) variant IDVA.



(a)



(b)

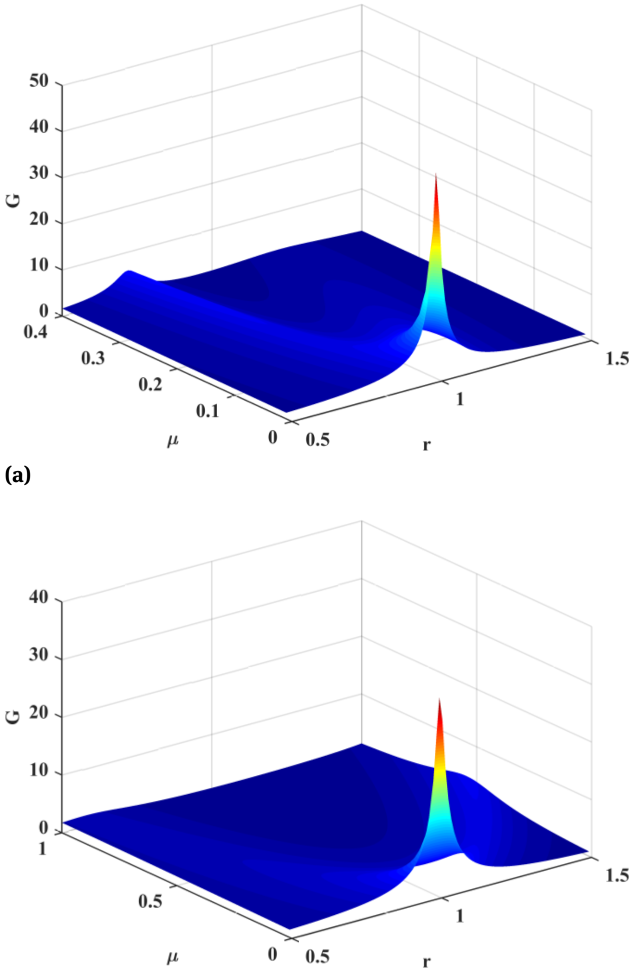
Figure 5: Comparison of amplitude magnification factor of primary system: (a) traditional IDVA, (b) variant IDVA for different values of inerter-to-mass ratio (δ).

choosing a proper inertance-to-mass ratio, frequency and absorber damping ratio by adding inerter alone to the variant DVA. Also, Figure 5(b) clearly shows that the frequency bandwidth of variant IDVA is wider than traditional IDVA for particular value of design parameters.

The proposed minimax approach is also used to study the effect of mass ratio on the optimal amplitude magnification factor of primary system in traditional and variant IDVAs over the frequency range as shown in Figures 6(a) and 6(b). The amplitude magnification factor of variant IDVA is small when compared to traditional IDVA for various values of mass ratio.

The optimal maximum amplitudes of primary system are plotted in Figure 7(a) over the range of $0 < \mu \leq 0.2$. As shown in Figure 7(b), the proposed IDVA (adding inerter alone to the variant IDVA) gives more vibration suppression (over 19%) than the traditional IDVA when the mass ratio is less than 0.2 for the following: $\delta = 0.1$, $\xi_1 = 0$. It is more than sufficient to conclude that the variant IDVA gives a better vibration suppression evidenced by a smaller G value, if the frequency, damping and inertance-to-mass ratios of the proposed IDVA are chosen properly.

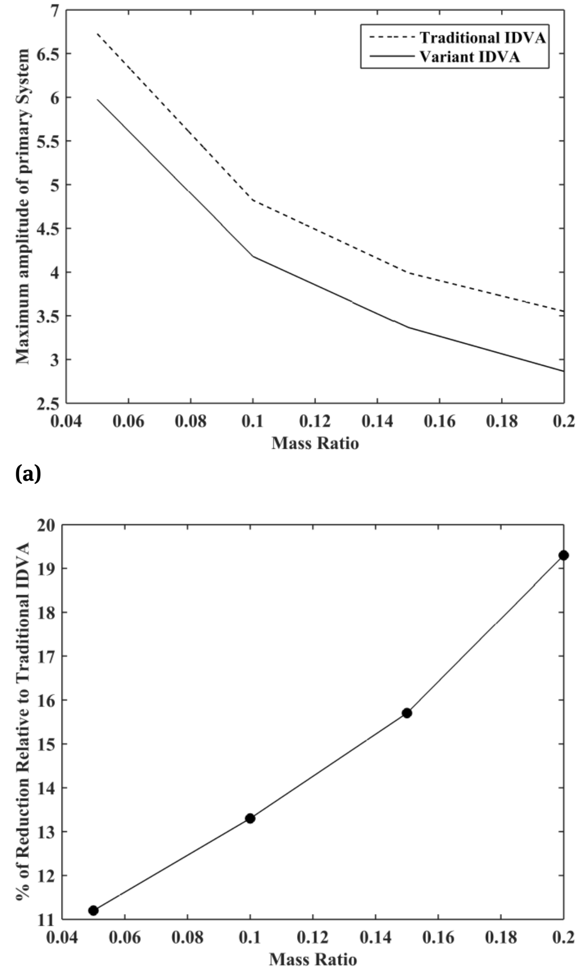
The amplitude magnification factors of traditional and variant IDVAs of undamped primary system excited by ground motion under optimum tuning and damping with were calculated analytically and the calculation result is plotted in Figure 8. It proved analytically that the proposed variant IDVA provides a larger suppression of resonant vibration amplitude of the primary system excited by ground motion than the traditional IDVA and also the frequency bandwidth is increased in the proposed variant IDVA.



(a)

(b)

Figure 6: Contour plot of amplitude magnification factor of primary mass with different mass ratios: (a) traditional IDVA, (b) variant IDVA when $\delta = 0.1$, $\xi_1 = 0$.



(a)

(b)

Figure 7: (a) Optimal maximum amplitude of primary system in the H_∞ optimization with different mass ratios when $\delta = 0.1$, $\xi_1 = 0.1$; (b) percentage reduction relative to traditional IDVA.

4 Conclusions

In this paper, the performance of inerter-based variant DVAs subjected to base excitation has been investigated, where the proposed model is obtained by adding an inerter alone in the variant DVA in parallel manner. Also, a closed-form analytical solution for the optimal parameters of the proposed variant IDVA, coupled with undamped primary system, is obtained by using fixed-points theory. The proposed IDVA and the traditional IDVA were compared and analytical results of the proposed variant IDVA exhibit frequency and damping ratios, as a function of the mass and inertance-to-mass ratio, higher than those of traditional IDVA optimal parameters. The results showed that adding inerter alone parallel to the variant DVA provided 19% improvement in the case of vibration reduction obtained over

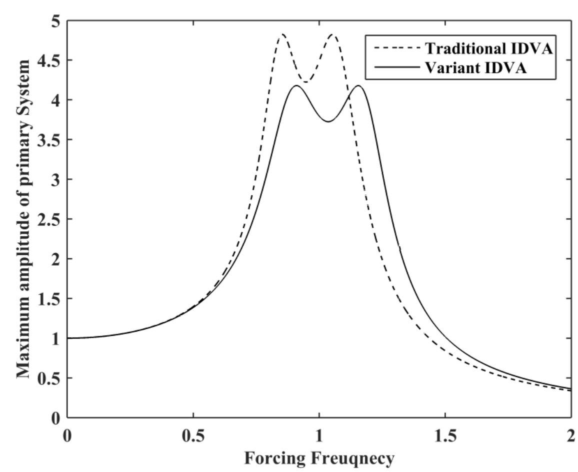


Figure 8: Comparison of amplitude magnification factor of primary system.

traditional IDVA when mass ratio was less than 0.2. It has also been observed that when the inertance-to-mass ratio increases, the amplitude magnification factor of the primary system also increases in traditional damped IDVA, whereas it decreases in the proposed IDVA. Furthermore, the effect of absorber mass ratio on the amplitude magnification factor of the primary system was studied over the frequency range. Finally, it is also observed that for the same model parameters, the frequency bandwidth of the proposed IDVA is wider than that of traditional IDVA.

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